ANALYSIS AND MEASUREMENT OF POVERTY AND SOCIAL EXCLUSION USING FUZZY SET THEORY. APPLICATION AND POLICY IMPLICATIONS.

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September 2002

ABSTRACT

This research presents and discusses the relative merits and limitations of the univariate and multivariate analyses and measurement of poverty to represent the state of poverty, the poverty ratio time path, and to assess the power of these approaches to identify the main causes of poverty and to inspire the proposal of sound socioeconomic policies.

The univariate measurement of poverty analyses and estimates simple and composite poverty ratios advanced in the literature and their limitations to represent observed poverty time path and the lack of structural socioeconomic policy implications.

The multivariate analysis of poverty advances forward the French social exclusion theory, and Sen's analysis of functioning and capability, making them operational, in the sense of providing a poverty ratio and deriving its policy implications. In effect, to these analyses are applied the fuzzy set theory to obtain: (i) the poverty ratio of each household; (ii) the poverty ratio of a population of households; and (iii) very important for its policy implications, the poverty ratio of the population by retained attribute, such as, years of schooling of the household head and spouse (if present), house size and condition, and house endowment of sanitary and other services (drinking water, bath, electricity, etc.). Point (iii) allows the researcher to identify the main causes of structural poverty, i.e., the lack of those attributes that contribute to reproduce poverty from generation to generation.

The outcome of this research is applied to the data base provided by the Bank of Italy sample surveys in 1993, 1995, 1998 and 2000, and a comparative analysis of the uni- and multi-variate approaches to the measurement of poverty and their policy implications for Italy completes this study.

1. INTRODUCTION

The concept of poverty, together with poverty line and the head-count ratio as a relative measure of poverty, were first introduced, over a century ago, by Booth (1892) and Rowntree (1901). These contributions inspired further theoretical, empirical and methodological development. Introducing the basic needs approach, whose main components were food, housing and clothing, they arrived at the construction of a poverty line and an estimation of poverty. The poverty line purports to capture the adopted concept of poverty and to become a rigid discriminant line that separates the poor from the non-poor.

Since the 1970s, the line of research pioneered by Booth and Rowntree evolved in several directions. One of them acquired a special momentum with the publications of a seminal paper by Sen (1976), where he included, in a single scalar measure of poverty, the head-count ratio, the poverty gap ratio, *i.e.*, the intensity of poverty, and the Gini income inequality ratio among the poor. This direction of research is indeed a "univariate approach" (UA) because it considers a single dimension, generally income, sometimes expenditure, as the only variable retained to capture the concept of poverty as an insufficient command over resources.

Contemporarily, multivariate approaches (MA) started to be developed with the scope of achieving a more comprehensive analysis and measurement of poverty.

The view of poverty as multiple deprivation enriched the explanatory power of this field of research. Moreover, by identifying the dominant dimensions of poverty, it provided the basic information for the design and implementation of structural socioeconomic policies purporting to generate socioeconomic processes to reduce the relative proportion of poor as well as the intensity of poverty.

The main multivariate analysis of poverty developed in the last three decades are:

(i) the *social exclusion* approach, first introduced by the French Minister of Social Welfare René Lenoir, in 1974;

(ii) the *functioning, capability* and *entitlement* approaches introduced and analyzed by Sen (1980, 1981, 1982*a*, 1982*b*, 1985, 1992, 1993);

(iii) the *fuzzy set* approach to the analysis and measurement of poverty introduced by Cerioli and Zani (1990) and further discussed by Dagum *et al.* (1992), Cheli *et al.* (1994), Cheli and Lemmi (1995), Martinetti (1994); see also Delbono (1984, 1989);

(iv) the UNDP Human Poverty Index (HPI) presented in its Human Development Report (1997, 1998). In 1991, the UNDP introduced a Human Development Index (HDI) to measure the relative levels of development among countries. The HDI is a function of the following indicators: (a) income per capita adjusted for purchasing power parity; (b) life expectancy at birth; (c) primary, secondary and tertiary enrolment ratio; and (d) adult literacy ratio.

Supported by similar socioeconomic foundations leading to the specification of the HDI, the UNDP (1997, 1998) introduced two versions of a human poverty index; one for developing countries (HPI-1), and the other for industrialized countries (HPI-2). The former, HPI-1, considered deprivation in three essential dimensions in human life, *i.e.*, longevity, knowledge, and a human standard of living. The latter, HPI-2, considered deprivation in four essential dimensions in human life; namely, the three already introduced in HPI-1, and the degree of deprivation for a lack of social inclusion.

Van Praag (1978) introduced a new approach to the measurement of poverty based on sample surveys about the perception of poverty of the interviewed persons. This approach lead not to the perceived poverty index but rather to the perceived relative personal welfare of the interviewed; it was strongly conditioned by the welfare levels of the members belonging to his/her professional group. In international and inter-regional comparisons, the estimated index reflected the different levels of expectation and the *esprit bourgeois* of each population. A thorough discussion of this approach can be found in Hagenaars (1986) and a critical review of this in Dagum (1989). This approach will not be further discussed in this study.

The main scope of this study is: (i) to analyze the univariate and multivariate approaches to poverty; (ii) to propose the fuzzy set theory as a rigorous and efficient method for the analysis and measurement of poverty, including its power to operationalize both the social exclusion and Sen's functioning, capabilities and entitlement approaches; (iii) to discuss the policy implications of the univariate and multivariate approaches; and (iv) to illustrate the development presented with a case study.

This research is organized as follows: Section 2 presents some thoughts on the absolute and relative poverty lines and the measurement of poverty; Section 3 proposes, for the univariate approach, a methodological research program (MRP) for the analysis and measurement of poverty composed of eight steps, where the last one deals with its policy implications; Section 4 introduces a multivariate methodological research program for the analysis and measurement of poverty, including a discussion of its policy implications; Section 5 presents a method of statistical analysis to determine the degree of similarity in the identification of the poor households by the univariate and multivariate approaches; Section 6 applies the univariate and multivariate approaches to the analysis and measurement of poverty in Italy, for the years 1993, 1995, 1998 and 2000, using the sample surveys of the Bank of Italy on income and wealth distributions; and Section 7 presents the conclusions.

2. SOME THOUGHTS ON POVERTY ANALYSIS AND MEASUREMENT

Until the 1970s poverty has been dominantly an economic concept and dealt with personal (individual, family or household) levels of income or expenditure. For this, we call it a univariate approach (UA). In this context, poverty is defined as an insufficient command over resources for a person to be able to survive (absolute poverty) or to live according to the standard of living reached in the process of growth and development of a country (relative poverty), or something in between that would be partially but not totally sensitive to the per capita income (quasi absolute or quasi relative).

The absolute poverty approach considers the basic needs a person requires to survive, indeed, to physiological survive. The monetary value of the resources entering into the basic needs determines the poverty line of a population. It plays a central role in UA because it is used to discriminate between poor and non-poor persons. It is a strictly bivalent logic such that the population is partitioned into the poor and non-poor subsets (the *terzo escluso*).

In UA, the relative poverty concept is a function of the level of economic development and the historical and sociocultural characteristics of an economic space . This concept of poverty determines a poverty line as a function of the mean income of the population and the size of the households.

The distinction between absolute and relative poverty lines is a *relative* concept in the sense that it is defined as a function of the mean income elasticity of the poverty line. If the elasticity is equal to zero, the poverty line is called absolute because it does not change with the level of economic growth given by, *e.g.*, the per capita national income. Instead, a relative poverty line has a mean income elasticity equal to one because it changes in the same proportion than the per capita national income. When the elasticity takes values between zero and one, the poverty line is called quasi relative or quasi absolute.

Dagum (1989) has argued that all types of poverty lines are relative with respect to the standard of living, level of economic development, and historic and sociocultural circumstances, and the social philosophy of a society. Hence, the so called absolute poverty line is *relative* to the low level of economic development. Thus, the poverty line of a very poor country struggling to survive will be determined in an essential way by the basic needs approach, whereas the poverty line of a rich country will incorporate other needs such as elementary and junior high school education, health care, and recreational and cultural facilities. Montesquieu's classic contribution on *the spirit of the laws* cogently supports and substantiates this statement. *Mutatis mutandis*, we should say *the spirit of the poverty lines*, and Montesquieu's contribution applies without loss of continuity. In effect, Montesquieu (1748, T.II, p. 238) states that "the laws must be relative to the physiognomy of the country; to its climate, *i.e.*, burning or temperate; (…) to the religions of its inhabitants, to their inclinations, to their wealth, to their number, to their trade, to their customs, to their manners. Finally *they are related to each other* (…). I shall examine these relationships: together they constitute what one calls the spirit of the laws". The italics were added with the purpose of stressing Montesquieu's adoption on an explicit form of interpersonal comparisons of utility that only extreme ideological biased approaches would barren it from socioeconomic inquiries and policies.

The univariate approach to the analysis and measurement of poverty can be the outcome of:

(i) statistical and economic assumptions leading to the specification of a poverty line, equivalence scale and a poverty ratio. The latter stated as a function of one or more of the following indicators: head-count ratio, income gap ratio, directional economic distance ratio between poor and non-poor, income means, Gini ratio of the poor, and Gini ratio of the non-poor. Among the indexes belonging to this class we mention: the head-count ratio, income gap ratio, Sen (1976), Takayama (1979), Kakwani (1989), Thon (1979), Foster-Greer-Thorbecke (1984), Dagum-Lemmi-Cannari (1988) and Dagum-Gambassi-Lemmi (1992) indexes;

(ii) social welfare assumptions stated as a function of an utility function, such as the indexes advanced by Clark-Hemming-Ulph (1981), Blackorby-Donalson (1980), Pyatt (1990), Vaugham (1987), Chakravarty (1990), and Hagenaars (1986). The utility function specified by all of them is a strictly additive and separable function of the income of each economic unit, therefore it arbitrarily eliminates any interpersonal comparisons of utility, which is a very convenient and extreme mathematical simplification done at the cost of losing any meaningful representation of reality. Hence, they depart from the XVIII Century seminal contributions of Paley, Helvetius, Palmieri (Dagum, 1993), and Bentham (1789).

Unlimited and uncritical applications of the important Pigou-Dalton's principle of transfers, which is only valid when the observed income inequality is greater than a socially just (equitable) distribution, leads this School to the unacceptable conclusion that social welfare is maximized when income is equally distributed. For a further discussion of this issue, see Dagum (1995*a*, 2001*a*). Besides, the utilitarian school replaces the income of each economic unit, which is an observable variable, by a latent function of this variable, requiring only that it should be increasing and concave (decreasing marginal utility). It results in an important loss of information that will distort any meaningful welfare interpretation of a society. For this reason, this class of poverty measurement will not be further discussed.

3. A METHODOLOGICAL RESEARCH PROGRAM FOR A UNIVARIATE ANALYSIS AND MEASUREMENT OF POVERTY

In a comment of Hagenaars's (1986) book, Dagum (1989) proposes a methodological research program (MRP) for UA analysis and measurement of poverty and its implications for a socioeconomic policy purporting to reduce the extend, intensity and inequality of a poor population. Dagum distinguishes between the structural and the business cycle causes of poverty.

It follows a stepwise presentation of a MRP for UA research on poverty.

STEP 1. Identification of the population object of research, its fourfold philosophy of science interpretation, and the choice of the variable to determine the state of poverty. In general, the object of poverty research is a population of households. Thus, our economic units are households belonging to an economic space (nation or region), or subsets of this population, partitioned with respect to some socioeconomic attribute such as gender, years of schooling, urban-rural and age. It is our sample space. Symbolically,

(1) $$\mathbf{A} = \{\mathbf{a_1},...,\mathbf{a_i},...,\mathbf{a_n}\},$$

where **n** is the cardinality of the set **A**, therefore, we are dealing with **n** observed households. In the case of a census, **A** contains all the households of a population, hence, each $\mathbf{a_i} \in \mathbf{A}$ has the constant weight of **1**, **i=1,...,n**. If **A** is a representative sample of a population, being it a stratified sample, which includes representative subsamples of some socioeconomic attributes of the household head, to each $\mathbf{a_i}$ corresponds a weight $\mathbf{n_i}$ equal to the number of households the sample observation $\mathbf{a_i}$ represents, and $\sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{n_i} = \mathbf{N}$, the size of the population. Its relative frequency is $\mathbf{n_i / N}$.

To the **i**-th household is associated an income variable $\mathbf{Y_i}$, such that,

(2) $$\mathbf{Y_i} = \mathbf{Y(a_i)},\ \mathbf{i = 1,...,n}.$$

Dagum (1995*b*, 2001*a*) presents the following fourfold philosophy of science interpretation of the set **A**.

(i) **Semantic of science interpretation.** The set **A** is the *factual referent* for the analysis and measurement of poverty, and constitutes the essential referent set when dealing with the *factual sense* and the *factual truth* in the assessment of the basic assumptions and final propositions of a research on poverty. For a philosophy of science discussion of semantic of science, see Bunge (1974, Vols. I and II).

(ii) **Ontology of science interpretation.** The set **A** contains the essential elements, *i.e.*, the *being*, whose nature and ground is the object of any methodological research program on poverty.

(iii) **Methodology of science interpretation.** The set **A** is a *sample space*. Hence, from **A** and the specification of an appropriate class **G** of subsets of **A** as a base for the topology **T(G)**, we generate all the subsets of **A** relevant to the scope of a research. In a research on poverty, a natural base is the *discrete partition* of **A**, hence **T(G)** becomes the power set **P**(**A**). One of its members is the subset $\mathbf{B} \subset \mathbf{A}$, such that, $\mathbf{B} \in \mathbf{P}(\mathbf{A})$, where **B** is the set of poor households.

The power set $\mathbf{P}(\mathbf{A}) = \Gamma$ is a sigma algebra. Assigning to each member $\gamma \in \Gamma$ a probability $\mathbf{P}_\gamma$, it is obtained the probability space (**PS**)

(3) $$(\mathbf{A}, \mathbf{P}(\mathbf{A}), \mathbf{P}_\gamma) = (\mathbf{A}, \Gamma, \mathbf{P}_\gamma).$$

The income variable **Y** in (2) maps the **PS** in (3) into the induced **PS**

(4) $$\mathbf{Y: (A, \Gamma, P_\gamma) \rightarrow (R^+, B^+, P_\beta)},$$

such that , $\mathbf{R}^+$ is the non-negative orthant of real numbers (should we have negative income values, we should map **A** into the set of real numbers **R**); $\mathbf{B}^+$ is a Borel set and $\beta \in \mathbf{B}^+$, hence, $\beta \subset \mathbf{R}^+$ and $\mathbf{P}_\beta = \mathbf{P}_\gamma$, since **Y** maps the event γ into the event β.

The induced probability space generated by the income variable **Y** cogently introduces, as core methods of socioeconomic research on poverty, the content of probability theory, stochastic processes, statistical inference and methods of estimation. In UA, to the probability measure $\mathbf{P}_\beta$, $\beta \in \mathbf{B}$, corresponds the income distribution function

(5) $$\mathbf{F(y) = P(Y \leq y) = \int_0^y dF(\xi)}.$$

A mathematical specification of **F(y)** corresponds to a model of income distribution. For a systemic approach to the generation of income distribution models, see Dagum (1983, 1990, 1996, 2001b).

(iv) **Epistemology of science interpretation.** A research on poverty that starts with the specification of the set **A** of households and a set of basic assumptions, and ends with a set of theoretico-empirical propositions, has to be assessed with respect to the validity of its assumptions and propositions, and its limits. Hence, its study the factual sense of **A**, its associated **PS**, the income variable **Y**, its respective induced **PS**, and the factual truth of the deduced theory, model and poverty measure, and the applied scientific methods. Therefore, the set **A** has an epistemological interpretation, since epistemology is the study of the nature and ground of a scientific explanation and the scientific method applied to arrive at it, and the discussion of their limits and validity.

STEP 2. The adopted concept of poverty. The economic interpretation of poverty as an insufficient command over resources implies the adoption of an univariate approach to the analysis and measurement of poverty. Hence, the choice of income, or expenditure, as an indicator of command over resources, implies a dichotomic partition of the households population into *poor* and *non-poor*.

The choice of income **Y** as an indicator of command over resources, the set of households **A** such that the income of the **i**-th household is $\mathbf{y_i = y(a_i)}$, and the specification of non-overlapping income intervals in $\mathbf{R}^+$ corresponding to each $\mathbf{a_i} \in \mathbf{A}$ allow us to obtain the induced probability space (4) and the income distribution function (5).

STEP 3. Specification of a poverty line Z. This is a main step in the univariate approach, while it is a derived proposition in the multivariate approach.

UA deals with a single variable, let us say income. Therefore, for a representative household, *e.g.*, a two-adult household, the UA has to determine the level of income **Z=Z(2)**, *i.e.*, its poverty line. This poverty line will discriminate among poor and non-poor two-adult households. Very often, the literature erroneously calls them poor and rich households, respectively.

Given a two-adult household with income **y=Z-**ε, where ε is an infinitesimal, and another with null income, each of them is counted as a poor household. On the other hand, a two-adult household with income **y=Z**+ε and another with income **y=1000 Z** are counted as two non-poor households. This is a very strong limitation of UA to the analysis and measurement of poverty and its policy implications.

The two-adult household poverty line **Z(2)** has to be extended to households of any size **N**, *i.e.*, we have to determine what will the poverty line be for households of size 1, 3, 4, … . In other word, we have to transform the income of each household of size **N ≠ 2** into its equivalent income corresponding to the assumption that it is of size **N = 2**. Since the command over resources for a given level of welfare is not proportional to the household sizes, *e.g.*, **Z(4) ≠ 2 Z(2)**, we expect economies of scale, hence, **Z(3)** will be less than **1.5 Z(2)** and **Z(4)** will be less than **2 Z(2)**. The next step deals with this issue.

STEP 4. Equivalence scale. To pass from the income (usually, disposable income) of a household of any size to its equivalent level of income corresponding to the assumption that all households are of size two, we need to build an equivalence scale. It will also allows us to determine the poverty line of households of size $\mathbf{N \neq 2}$.

An approach to build an equivalence scale starts with the specification of an extended Engel microeconomic food consumption function, *i.e.*, food expenditures **FE** of a household as a function of its income **y** and size **N**. Hence,

$$\mathbf{FE = by^{\alpha}N^{\beta}}, \quad \mathbf{b > 0}, \quad \mathbf{0 < (\alpha, \beta) < 1}. \tag{6}$$

The constraint $\mathbf{0 < \alpha < 1}$ is supported by a well established behavioral regularity for cross section data which states that, given **N** constant, food expenditure increases with income at a decreasing rate. On the other hand, the constraint $\mathbf{0 < \beta < 1}$ implies that there is economy of scale, *i.e.*, given **y** constant, food expenditure increases with the size **N** of a household but at a decreasing rate. It follows from (6) that α and β are the partial elasticities of food expenditure with respect to income **y** and household size **N**, respectively.

Dividing eq. (6) by **y**, and postulating that all households with the same food expenditure – income ratio have the same level of welfare, we have

$$\mathbf{FE / y = by^{\alpha - 1}N^{\beta}}, \tag{7}$$

and for $\mathbf{FE / y =}$ constant, *i.e.*, an isowelfare or isoquant of (7), the elasticity of **y** with respect to **N** is,

$$\mathbf{e_{yN}} = \frac{\mathbf{d \log y}}{\mathbf{d \log N}} = \frac{\beta}{\mathbf{1 - \alpha}}. \tag{8}$$

The estimation of this elasticity plays an essential role in the construction of an equivalence scale. Since the welfare of a household is an increasing function of its income and a decreasing function of its size, we need to estimate **Z** for households of different sizes. This requires the construction of an equivalence scale **S(N), N = 1, 2, …**, where **S(N)** is an index number such that $\mathbf{S(N^*) = 100}$, where, in the case of industrialized countries, $\mathbf{N^* = 2}$, and frequently, for developing countries, $\mathbf{N^* = 4}$.

Therefore, given S(N*), we need to obtain the equivalence scale **S(N)** for all $\mathbf{N \neq N^*}$.

It follows from the identity

$$\frac{\mathbf{S(N + \Delta N) - S(N)}}{\mathbf{S(N)}} = \frac{\mathbf{\Delta S(N)}}{\mathbf{S(N)}}, \tag{9}$$

and eq. (8), that

(10)
$$\mathbf{S(N+\Delta N)=S(N)\left[1+\frac{\Delta S(N)}{S(N)}\right]=S_N\left[1+\frac{N}{S(N)}*\frac{\Delta S(N)}{\Delta N}*\frac{\Delta N}{N}\right]=}$$
$$\mathbf{=S(N)\left[1+e_{yN}\frac{\Delta N}{N}\right]}$$

therefore,

(11)
$$\mathbf{S(N+\Delta N)=S(N)\left[1+\frac{\beta}{1-\alpha}\frac{\Delta N}{N}\right]}.$$

It follows from (11), and given $\mathbf{N^* = 2}$, hence $\mathbf{S(2) = 100}$, that

$$\mathbf{S(1)=S(2)\left[1-\frac{\beta}{2(1-\alpha)}\right]=100\left[1-\frac{\beta}{2(1-\alpha)}\right]};$$

$$\mathbf{S(3)=100\left[1+\frac{\beta}{2(1-\alpha)}\right]};$$

$$\mathbf{S(4)=S(3)\left[1+\frac{\beta}{3(1-\alpha)}\right]};$$

and so on. Then, given $\mathbf{Z(N^*)}$, the poverty line $\mathbf{Z(N)}$ is

(12)
$$\mathbf{Z(N)=\frac{S(N)Z(N^*)}{S(N^*)}=\frac{S(N)Z(N^*)}{100}},\qquad \mathbf{N=1,2,...}\ .$$

The equivalence scale (11), besides being used to obtain from the poverty line $\mathbf{Z(N^*)}$ the poverty line $\mathbf{Z(N)}$ for any household of size $\mathbf{N \neq N^*}$, allows us to transform the income $\mathbf{y(N)}$ of an $\mathbf{N}$-size household into its equivalent income $\mathbf{y^e{=}y^e(N^*)}$, as it would be an $\mathbf{N^*}$-size household. Hence, for the **i**-th household,

(13)
$$\mathbf{y_i^e=y_i^e(N^*)=\frac{100y_i(N)}{S(N)}},\qquad \mathbf{i=1,2,...,n}\,.$$

STEP 5. Identification of the poor. Any **N**-size household with income $\mathbf{y(N) < Z(N)}$, or equivalently, $\mathbf{y^e(N^*) < Z(N^*)}$ is defined as poor, where $\mathbf{Z(N)}$ is given by (12), and $\mathbf{y^e(N^*)}$ is deduced from (13).

The total number $\mathbf{q \leq n}$ of households with income $\mathbf{y(N) < Z(N)}$, for all **N**, is the number of poor households in a population of size **n**.

STEP 6. The measurement of poverty. For the UA, the former steps provide all the necessary information to propose a poverty measure of a given population of households. The literature on this argument is very rich and some well known proposed poverty indexes have been mentioned above.

Let us now consider the most essential features that should enter in an UA to achieve a comprehensive representative poverty measure of societies composed of poor and non-poor households.

For a population of **n** households, it follows from Step 5 that they are **q** poor and **n – q** non-poor households, and from (13) the corresponding equivalent household incomes $\mathbf{y^e(N^*)}$ were obtained. These data allow the proposition of useful indicators to advance synthetic UA measures of poverty. They are:

6.1. Diffusion or Head-count ratio H. By definition, it is the proportion of poor households in the total population of households, *i.e.*,

(14)
$$\mathbf{H = \frac{q}{n} = F\big(Z(N^*)\big) = P\big(y^e(N^*) < Z(N^*)\big) = \\ = P\big(y(N) < Z(N);\qquad N = 1,2,...\big).}$$

6.2. Intensity of poverty or Income gap ratio I. It is defined as the ratio between the average income gap of the poor and its poverty line, *i.e.*,

(15)
$$\mathbf{I = \frac{\sum_{i=1}^{q} g_i^e}{qZ(N^*)} = \frac{\sum_{i=1}^{q}\left[Z(N^*) - y_i^e(N^*)\right]}{qZ(N^*)} = \\ = 1 - \frac{\bar{y}_p^e}{Z(N^*)} = 1 - \frac{\bar{y}^e L(Z(N^*))}{Z(N^*)F(Z(N^*))}},$$

where $\mathbf{\bar{y}_p^e}$ stands for the average equivalent income of the poor, $\mathbf{\bar{y}^e}$ for the average equivalent income of the households population, and $\mathbf{L(Z(N^*))}$ and $\mathbf{F(Z(N^*)) = H}$ for the Lorenz curve and the cumulative distribution of equivalent income, respectively. For the household equivalent incomes, the income gap is defined as follows:

(16)
$$\mathbf{g_i^e = Z(N^*) - y_i^e(N^*),\quad \forall\, y_i^e(N^*) < Z(N^*),\quad i = 1,2,...,n}\,;$$

and **0** otherwise. For the household observed incomes, it follows from (12), (13) and (16), that

(17)
$$\mathbf{g_i(N) = Z(N) - y_i(N),\quad \forall\, y_i(N) < Z(N),\quad N = 1,2,...;\quad i = 1,2,...,n}\,.$$

6.3. Income inequality of the poor $\mathbf{G_p}$ and the non-poor $\mathbf{G_{np}}$ households. The poverty line $\mathbf{Z(N^*)}$ partitions the population into **q** poor and **n – q** non-poor households. For poverty analysis, and for the design of a socioeconomic policy, it is important to know the income inequalities of the poor and the non-poor subpopulations. Ordering the **q** poor and the **n – q** non-poor households by the increasing size of their corresponding equivalent incomes and using the well known Gini ratio, we have, for each of these two subpopulations,

(18) $$G_p = \frac{\Delta_p^e}{2\bar{y}_p^e} = -1 - \frac{1}{q} + \frac{2\sum_{i=1}^{q} i y_i^e}{q^2 \bar{y}_p^e},$$

and

(19) $$G_{np} = \frac{\Delta_{np}^e}{2\bar{y}_{np}^e} = -1 - \frac{1}{n-q} + \frac{2\sum_{i=1}^{n-q} i y_{q+i}^e}{(n-q)^2 \bar{y}_{np}^e},$$

where Δ_p^e and Δ_{np}^e stand for the Gini mean differences of equivalent incomes of the poor and the non-poor households, respectively.

6.4. Directional income distance ratio D between the poor and the non-poor households. It purports to estimate the relative directional distance or relative deprivation of the poor with respect to the non-poor subpopulations of households. It is stated as a function of the poor and non-poor averages of equivalent incomes.

Given that these two subpopulations do not overlap, Dagum *et al*. (1992) proposed the following directional distance ratio **D**:

(20) $$D = \frac{\left(\bar{y}_{np}^e - \bar{y}_p^e\right)}{\left(\bar{y}_{np}^e + \bar{y}_p^e\right)}$$

It can be proved that

$$D = \frac{[F(Z(N^*)) - L(Z(N^*))]}{[F(Z(N^*)) + L(Z(N^*)) - 2F(Z(N^*))L(Z(N^*))]}.$$

STEP 7. Specification of an UA comprehensive poverty ratio. Rowntree (1901) specified the head-count ratio **H** as a poverty measure. It captures a very important aspect of poverty, *i.e.*, its diffusion. It says nothing about the intensity of the deprivation of being poor, the relative deprivation stemming from the income inequalities of the poor and non-poor, and on the disparity in mean of the two subpopulations.

Several authors, such as Kakwani (1980), Thon (1979) and Foster *et al.* (1984), proposed weighted averages of the income gap of the poor; Sen (1976) was the first to advance a comprehensive measure of poverty. Starting from a set of axioms he arrived to the poverty ratio,

(21) $$P_s = H\lfloor I + (1-I)G_p \rfloor.$$

Sen combines in a single measure, (*i*) the head-count **H**, (*ii*) the income gap **I**, and (*iii*) the Gini income inequality G_p ratios of the poor, as defined in (14), (15) and (18), but ignores the directional distance between the income means of the poor and non-poor and the income inequality of the non-poor, as if the poor and non-poor were not members of the same society.

Takayama (1979) purports to go beyond Sen index incorporating the income of the non-poor in a more encompassing measure. However, he makes a counterfactual assumption that destroys the scope of his measure by assuming that all non-poor households have an income equal to the poverty line.

Dagum, Lemmi and Cannari (1988) and Dagum, Gambassi and Lemmi (1992) extend Sen index introducing the Gini ratio of the non-poor given in (19), and the relative deprivation in mean of the poor with respect to the non-poor subpopulations given in (20).

Dagum, Gambassi and Lemmi combine the five ratios discussed in Step 6 and formalized in (14), (15), (18), (19) and (20), proposing the following comprehensible poverty measure:

$$\mathbf{P_{DGL}} = \mathbf{P}(\mathbf{H}, \mathbf{I}, \mathbf{D}, \mathbf{G_p}, \mathbf{G_{np}}) = \mathbf{H}(\mathbf{I} + \mathbf{D} + \alpha|\mathbf{G_p} - \beta\mathbf{G_{np}}|), \quad \alpha > \mathbf{0}, \quad \mathbf{0.5} \le \beta \le \mathbf{0.8}. \tag{22}$$

Besides being $\mathbf{P_{DGL}}$ a comprehensive measure of poverty, stated as a function of $\mathbf{H}$, $\mathbf{I}$ and $\mathbf{G_p}$, it provides important insights on the diffusion and the intensity of poverty and its disparity among the poor, and being also a function of $\mathbf{D}$ and $\mathbf{G_{np}}$, it provides essential insights on the relative income deprivation and income disparity of the poor with respect to the non-poor.

STEP 8. Socioeconomic analysis of the UA research outcome and policy implications. The univariate approach provides little information for the design of a socioeconomic policy to steadily reduce poverty. In effect, it does not provide useful information for the design of a structural economic policy aimed at a steady extirpation of the causes of poverty. It provides information to activate policies tending to alleviate the state of poverty through government transfers and non-government organized social services but it is totally incapable of dealing with the causes of poverty. The transfers of resources have a legitimate place as a part of a countercyclical economic policy and for a temporally relief of the poor, otherwise they are a source of humiliation and contribute to aggravate their sense of social exclusion.

The most informative univariate approach to the measurement of poverty is $\mathbf{P_{DGL}}$ specified in (22). In effect, each of the five components entering into the definition of $\mathbf{P_{DGL}}$ and its synthetic index contain useful information for the design of a countercyclical socioeconomic policy and for a temporally activation of a policy of transfers.

A more exhaustive and informative measurement of poverty has to be the outcome of a multidisciplinary and multivariate analytical framework capable of identifying and measuring the main causes that contribute to the observed state of poverty. Hence, it should provide the necessary insights for the design and activation of a structural socioeconomic policy aimed at the steady abatement of the causes of poverty. The analytical framework has to be enriched with the contributions of the European social exclusion school and Sen's capability and entitlement approaches. The measurement of poverty and its policy implications can be strongly enhanced by the application of the fuzzy set theory (see Zadeh, 1965 and the excellent textbook by Dubois and Prade, 1980). Section 4 deals with these topics.

4. A MULTIVARIATE APPROACH TO THE ANALYSIS AND MEASUREMENT OF POVERTY

In the MA, a rigorous and comprehensive analysis and measurement of poverty can be achieved applying the fuzzy set theory. Besides, it will be substantiated that the application of this theory provides basic information for the design of socioeconomic policies addressed to the gradual elimination in time of the causes that produce and reproduce intergenerational states of poverty.

Some steps introduced in the MRP proposed in Section 3 for the UA analysis and measurement of poverty are not longer applicable to MA.

A proposed MRP for MA follows.

STEP 1. Identification of the population object of research. As in UA, the population object of inquiry is the set **A** presented in (1).

STEP 2. The MA concept of poverty. A multivariate concept of poverty demands a multidisciplinary analysis. Three main socioeconomic conceptual developments were introduced in the last three decades, although the first two were not made operational. The first one is the more embracing concept of social exclusion. It was introduced in 1974 by the French Government Minister of Social Welfare René Lenoir. It has a strong mixture of individual and social dimensions, and it became a very fruitful and stimulating field of research in continental Europe and the third world. The second one was introduced by Sen (1985) and further developed in several other contributions of this author. In his analysis of poverty, Sen deals with the concepts of functioning, capabilities and entitlement.

Although the social exclusion approach is more socially oriented than Sen's, they are closely related and in need of a quantitative operationalization to be able to offer a meaningful and representative measurement of social exclusion and poverty.

Room (1992, p. 14) portrays social exclusion in relation to the social rights of citizens, to a certain basic standard of living and to the participation in the mayor social and occupational opportunities of the society. It purports to "study the evidence that, where citizens are unable to secure their social rights, they will tend to suffer processes of generalized and persisting disadvantage and their social and occupational participation will be undermined" (see also Gore, 1995, p. 2).

Unlike income or expenditure as the only variable considered in the UA, the social exclusion approach introduces and analyzes a vector of variables and attributes retained as indicators of some form of deprivation or poverty.

Gore (1995, p. 3) observes that “interest in social exclusion has grown in western Europe in relation to rising rates of unemployment, increasing international migration, and the dismantling, or cutting back, of welfare states”. In effect, labor market segmentation and informalization, the fiscal crises that government use as an excuse to restrict the universality of social coverage and the dominantly unidirectional international migration, as an aftermath of the reverse unidirectional imperialist occupation of the past, have brought to the fore issues such as race and ethnic relations, citizenship, nationality, and long term unemployment and underemployment in multicultural and multiracial societies.

Research on social exclusion identifies a long list of economic and social phenomena. Among them, Silver (1995, pp. 74-75) includes:

(a) long term and recurrently unemployed;
(b) employed in precarious and unskilled jobs;
(c) low-paid worker and the poor;
(d) the landless;
(e) the unskilled, illiterate, and school dropouts;
(f) mentally and physically handicapped and disabled;
(g) substance abuses;
(h) child laborers and other forms of children abuse;
(i) racial, linguistic, ethnic and religious minorities;
(j) the political disenfranchised;
(k) recipients of social assistance;
(l) those needing, but ineligible for, social assistance.

Sen’s approach considers a person endowment of commodities, including the characteristic vector of those commodities, and the mores and customs of the society to which that person (individual, family, household) belongs. Then he presents the concepts of functioning as a person space of possible actions, and of capability as that person ability to optimize the use or the consumption of the commodity endowment. Hence, capability is a person ability to be or to do something.

The UNDP (1997, 1998) developed a third multivariate approach to the analysis and measurement of poverty; its annual Human Development Report publishes two Human Poverty Indexes, one for developing countries (HPI-1) and another for industrialized countries (HPI-2).

Unlike the social exclusion and Sen's approaches, the UNDP approach was made operational because it ends with a proposed measure of poverty.

The HPI for developing countries is an average of percentages of deprivation in three essential dimensions of human life. Since 1991, these dimensions were already included in the UNDP Human Development Index (HDI).

The component of the HPI-1 are:

(i) deprivation in longevity; it is estimated by the percentage of people not expected to survive to age 40;

(ii) deprivation in knowledge; it is estimated by the percentage of adults who are illiterate; and

(iii) deprivation in living standard; it is estimated by an arithmetic mean of, (a) the percentage of people without access to safe water; (b) the percentage of people without access to health services; and (c) the percentage of moderately and severely underweight children under five.

The HPI-1 for developing countries is the potential mean of order three of these three percentages.

On the other hand, the HPI-2 for industrialized countries is the potential mean of order three of the percentages of deprivation in four essentials dimensions of human life, *i.e.*,

(i) deprivation in longevity, estimated by the percentage of people not expected to survive to age 60;

(ii) deprivation in knowledge, estimated by the percentage of people who are functionally illiterate as defined by the OECD;

(iii) deprivation in standard of living, estimated by the percentage of people living below the income poverty line, set at 50% of the median disposable household income; and

(iv) deprivation because social exclusion or non-participation, estimated by the rate of long term unemployment of the labor force, *i.e.*, unemployment lasting 12 or more months.

The two HPIs proposed by the UNDP are meaningful indexes of poverty. They provide some insights to support the design of efficient socioeconomic policies to reduce poverty and social exclusion. There is room to refine the estimations of the deprivation percentages. Besides, the average of the percentages of deprivation should be the geometric mean because we are averaging percentages; a second best would be the arithmetic mean. The UNDP choice of the potential mean of order three is unsustainable. It significantly overestimates the poverty index. In effect, given at least two different positive quantities, Dagum (1979) proved that the potential mean of order **r**, **r** real, is a monotonic increasing function of **r**. Being the geometric mean the potential mean of order zero, it is smaller than the potential mean of order three, and in the case of the UNDP human poverty index, it can be verified that the geometric mean is significantly smaller than HPI-1 and HPI-2.

STEP 3. Choice of the set of socioeconomic attributes related to the state of poverty. Based on the information available, *e.g.* a sample survey or a census, we select the socioeconomic attributes whose lack of, or partial (insufficient) possession of any of those attributes, contributes to the state of a household poverty. They are represented by the **m**-order vector of attributes

(23) $$\mathbf{X} = (\mathbf{X_1,...,X_j,...,X_m})$$

The **m**-order vector **X** maps the probability space (3) into a new PS, *i.e.*

(24) $$\mathbf{X : (A, \Gamma, P_\gamma) \to (R_m^+, B^+, P_\beta)},$$

where $\mathbf{R_m^+}$ is the non-negative **m**-dimensional Euclidean space, $\mathbf{B^+}$ is a Borel set generated by a base of $\mathbf{R_m^+}$, and as in (4), $\mathbf{P_\beta = P_\gamma}$.

The multivariate distribution function of **X** is

(25) $$\mathbf{F(x_1,...,x_j,...,x_m) = P(X_1 \le x_1,...,X_j \le x_j,...,X_m \le x_m)}.$$

The **m**-order vector of attributes considered in a multivariate approach to the analysis and measurement of poverty includes economic, social, cultural, family and political attributes represented by continuous and discrete quantitative, and dichotomic and politomic qualitative variables. Among the **m** attributes considered in **X** we mention: income of the household; years of schooling of the household head (**H**) and the spouse (**S**), if present; age, job-status and gender of **H** and **S**; years of schooling of the parent of **H** and **S**; occupation of **H** and **S**; size, ownership and typology of the household residence; drinkable water; sanitary (bathroom, shower, sewage) services; and number of senior and handicapped persons in the household.

STEP 4. Equivalence scale. In MA the equivalence scale is needed to transform the income variable, entering as an element of the **m**-order vector ***X***, into equivalent levels of income for households of any size.

STEP 5. Specification of a poverty line. This is not a main step in MA; it is a derived proposition. Once we estimate the MA poverty index (MAPI), we make

(26) $$\mathbf{MAPI = H = F(Z) \Rightarrow Z = F^{-1}(MAPI) = F^{-1}(H)},$$

where **Z** is an imputed poverty line in MA. The symbol **H** stands for the head-count ratio, *i.e*. the percentage of households that are poorer than the average MAPI, and **F(y)** stands for the distribution of equivalent income.

STEP 6. Measuring poverty: the fuzzy set approach. A highly efficient and rigorous method to operationalize a multivariate analysis of poverty, including social exclusion and Sen's capability approaches, makes use of the fuzzy set theory. It purports to arrive at a poverty index as a function of the **m** attributes included in **X**. Cerioli and Zani (1990) applied fuzzy set theory to estimate the poverty in the Province of Parma (Italy). Dagum et al. (1992), Cheli et al. (1994), Cheli and Lemmi (1995) and Martinetti (1994) among others made further contributions and applications.

The fuzzy set theory allows to: (a) measure each household relative level of poverty or deprivation; (b) estimate the average poverty index of the population of households; and (c) measure the relative deprivation and poverty corresponding to each component or attribute included in **X**. The latter index is of a paramount importance for its policy implications. It identifies the most important variables or dimensions of poverty that need to be addressed to achieve a structural reduction of poverty, *i.e*., to implement a structural socioeconomic policy purporting to target institutional, behavioral, technological and social structural changes with the scope of generating dynamic economic processes of growth and development with less social exclusion, decreasing absolute and relative levels of poverty, and more equity.

Let **B** stands for a subset of households in **A** such that, any household $\mathbf{a_i} \in \mathbf{B}$ presents some degree of poverty in at least one of the **m** attributes included in **X**. Then, the subset **B** is a fuzzy subset of **A** and can be partitioned as follows (Fig. 1):

(27) $$\mathbf{B} = \{\mathbf{B}^*, \mathbf{B}^{**}\}, \qquad \text{s.t.,} \qquad \mathbf{B}^* \cup \mathbf{B}^{**} = \mathbf{B}, \qquad \mathbf{B}^* \cap \mathbf{B}^{**} = \varnothing$$

where $\mathbf{B}^*$ is the subset of households that are totally poor in all of the **m** attributes, and $\mathbf{B}^{**}$ the subset of households that are partially or totally poor in at least one attribute but not totally poor in all of them.

FIGURE 1

Let

(28) $$\mathbf{x_{ij}} = \mu_{\mathbf{B}}(\mathbf{X_j}(\mathbf{a_i})), \qquad \mathbf{0} \le \mathbf{x_{ij}} \le \mathbf{1}$$

stands for the degree of membership to the fuzzy set **B** of the **i**-th household (**i=1,...,n**) with respect to the **j**-th attribute (**j=1,...,m**), such that, (i) $\mathbf{x_{ij}=1}$, iff the **i**-th household does not possess the **j**-th attribute; (ii) $\mathbf{x_{ij}=0}$ iff the **i**-th household possesses the **j**-th attribute; and (iii) $\mathbf{0<x_{ij}<1}$ iff the **i**-th household possesses the **j**-th attribute with an intensity belonging to the open interval (0,1).

Let $\mu_B(\mathbf{a}_i)$ stands for the poverty ratio of the **i**-th household, *i.e.*, the degree of membership of the **i**-th household of the fuzzy set **B**. It is defined as the weighted average of $\mathbf{x_{ij}}$,

(29) $$\mu_B(\mathbf{a}_i) = \sum_{j=1}^{m} \mathbf{x}_{ij}\mathbf{w}_j \Big/ \sum_{j=1}^{m} \mathbf{w}_j \ ,$$

where $\mathbf{w_j}$ is the weight attached to the **j**-th attribute. It follows from the definition of $\mathbf{x_{ij}}$ in (i), (ii) and (iii) above, that,

(30) $$\mathbf{0} \le \mu_B(\mathbf{a}_i) \le \mathbf{1} \ ,$$

such that

(i) $\mu_B(\mathbf{a}_i) = \mathbf{0}$, iff $\mathbf{a}_i \in \mathbf{C} \setminus \mathbf{B}$, *i.e.*, $\mathbf{a_i}$ is completely non-poor in the **m** attributes, hence, $\mathbf{a}_i \notin \mathbf{B}$;

(ii) $\mu_B(\mathbf{a}_i) = \mathbf{1}$, iff $\mathbf{a}_i \in \mathbf{B}^*$, *i.e.*, $\mathbf{a_i}$ is totally deprived of the **m** attributes, hence, $\mathbf{a}_i \notin \mathbf{C}$;

(iii) $\mathbf{0} < \mu_B(\mathbf{a}_i) < \mathbf{1}$, iff $\mathbf{a}_i \in \mathbf{B}^{**}$, hence, $\mathbf{a}_i \in \mathbf{B} \cap \mathbf{C}$, *i.e.*, $\mathbf{a_i}$ is partially or totally deprived of some attribute but not totally deprived of all of them.

$\mu_B(\mathbf{a}_i)$ in (29) measures the degree of poverty of the **i**-the household as a weighting function of the **m** attributes. Hence, it measures the relative deprivation, degree of social exclusion, and insufficient capability of the **i**-th household to reach a living standard of the society to which it belongs.

The weight $\mathbf{w_j}$ attached to the **j**-th attribute stands for the intensity of deprivation of $\mathbf{X_j}$. It is an inverse function of the degree of deprivation of this attribute by the population of households. The smaller the number of households and the amount of their deprivation of $\mathbf{X_j}$, the greater the weight $\mathbf{w_j}$. For example, if one attribute is having safe drinkable water provided by a public utility service and the other is having a car no more than five year old, certainly fewer households will be deprived of the former and they will feel more intensively this deprivation.

Following Cerioli and Zani (1990, p. 277) we adopt a weight that fulfils the above property, *i.e.*,

(31) $$\mathbf{w}_j = \mathbf{log}\left[\sum_{i=1}^{n} \mathbf{g}(\mathbf{a}_i) \Big/ \sum_{i=1}^{n} \mathbf{x}_{ij}\mathbf{g}(\mathbf{a}_i)\right] \ge \mathbf{0}, \quad \sum_{i=1}^{n} \mathbf{x}_{ij}\mathbf{g}(\mathbf{a}_i) > \mathbf{0}, \quad \text{and} \quad \sum_{i=1}^{n} \mathbf{g}(\mathbf{a}_i) = \mathbf{n} \, .$$

The requirement that $\sum_{i=1}^{n} \mathbf{x}_{ij}\mathbf{g}(\mathbf{a}_i) > \mathbf{0}$ means that we do not consider an attribute $\mathbf{X_j}$ such that $\mathbf{x_{ij}=0}$ for all **i**. This would be an irrelevant attribute and should be excluded because there is not any deprivation in $\mathbf{X_j}$.

If the set **A** in (1) contains the total population of households, then $\mathbf{g(a_i)=1}$, and

(32) $$\mathbf{w_j = \log n \Big/ \sum_{i=1}^{n} x_{ij} \geq 0}.$$

The weight $\mathbf{w_j}$ in (31) and (32) is zero when $\sum_{i=1}^{n} \mathbf{x_{ij}} = \mathbf{n}$, *i.e.*, when the **j**-th attribute is not possessed by any of the **n** households, hence, $\mathbf{x_{ij}=1}$, **i=1,...,n**.

If the set **A** is a stratified sample of the population, the weight $\mathbf{g(a_i)}$ attached to the sample observation $\mathbf{a_i}$ is equal to **n** times the relative frequency of households in the total population. Hence, $\sum_{i=1}^{n} \mathbf{g(a_i) = n}$.

The fuzzy poverty index of **A** is a weighted average of $\mu_B(\mathbf{a_i})$ given in (29), hence,

(33) $$\mu_\mathbf{B} = \mathbf{P} = \sum_{i=1}^{n} \mu_\mathbf{B}(\mathbf{a_i})\mathbf{g(a_i)} \Big/ \sum_{i=1}^{n} \mathbf{g(a_i)} = \frac{1}{n} \sum_{i=1}^{n} \mu_\mathbf{B}(\mathbf{a_i})\mathbf{g(a_i)},$$

where $\mathbf{g(a_i)} \Big/ \sum_{i=1}^{n} \mathbf{g(a_i)}$ is the relative frequency represented by the sample observation $\mathbf{a_i}$ in the total population. For the particular case of a purely random sample or a census of **n** households, the weight is constant, and $\mathbf{g(a_i)} \Big/ \sum_{i=1}^{n} \mathbf{g(a_i)} = \mathbf{1/n}$, hence (33) becomes,

(34) $$\mu_\mathbf{B} = \mathbf{P} = \frac{1}{n} \sum_{i=1}^{n} \mu_\mathbf{B}(\mathbf{a_i}).$$

Given the household distribution of equivalent income $\mathbf{F(y^e)}$, the poverty line **Z(N*)** can be deduced, as in (26), from the fuzzy poverty index (33), and for a purely random sample, from (34). In effect, the imputed poverty line **Z(N*)** for equivalent households of size $\mathbf{N^*}$ is deduced from the equality

(35) $$\mu_\mathbf{B} = \mathbf{F(Z(N^*))}, \quad \textit{i.e.}, \quad \mathbf{Z(N^*) = F^{-1}(\mu_B)}.$$

STEP 7. Fuzzy set index $\mu_B(X_j)$ of the population for each attribute X_j, j=1,2,..,m. The fuzzy poverty index (33) and that of the **i**-th household (29) offer important information on the level and structure of poverty. In particular, unlike all the poverty indexes derived from UA, (29) estimates the relative degree of poverty of each household. Besides this clear additional advantage of the fuzzy set theory application to a multivariate analysis and measurement of poverty, it offers a further benefit since it contains the basic information that political decision makers need for the design of structural socioeconomic policies aimed at the steady abatement of the main causes of poverty and social exclusion. In effect, it follows from (29) and (33) that,

(36) $$\mu_B = \mathbf{P} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} w_j g(a_i)}{n\sum_{j=1}^{m} w_j} = \frac{\sum_{j=1}^{m}\left[\sum_{i=1}^{n} x_{ij} g(a_i) \Big/ n\right] w_j}{\sum_{j=1}^{m} w_j} = \frac{\sum_{j=1}^{m} \mu_B(X_j) w_j}{\sum_{j=1}^{m} w_j},$$

where,

(37) $$\mu_B(X_j) = \frac{1}{n}\sum_{i=1}^{n} x_{ij} g(a_i)$$

defines the degree of the **j**-th attribute deprivation by the **n** household population. The last member of (36) is the weighted average of (37), with weights w_j, **j=1,…,m**. It reproduces the fuzzy set poverty ratio μ_B presented in (33).

The analysis of the results obtained in (37), for **j=1,…,m**, allows the political decision makers to identify the most urgent areas of structural intervention, *i.e.*, an intervention addressed to a steady elimination of the causes of poverty and social exclusion that will be able to raise the poor households to the state of non-poverty, *i.e.*, to a secure and sustainable standard of living.

Unlike the policy of social assistance and transfers that are only palliatives for a temporary relief of the poor, creating an humiliating state of economic dependence, a structural economic policy purports the elimination of the causes of poverty, hence contributes also to the elimination of the intergenerational transmission of poverty that a policy of transfers is unable to accomplish.

STEP 8. α cuts and nested fuzzy sets. Another powerful policy implication brought about by the fuzzy set theory arises from the application of the α-cut and nested fuzzy set concepts. They allow the determination of nested subsets of poor households by their intensity of deprivation.

On this topic, we make a marginal modification of the textbook definition of α-cuts and nested fuzzy sets, to take into account that the state of poverty of the households is ordered by the decreasing values of their corresponding fuzzy set poverty ratios.

Given the set **A** of households and a fuzzy set $\mathbf{B} \subset \mathbf{A}$, an α-cut or level set is the fuzzy set $\mathbf{B}_\alpha$ such that,

(38) $$\mathbf{B}_\alpha = \{\mu_B(a_i) \geq \alpha, \quad a_i \in \mathbf{A}, \quad \alpha \in (0,1]\},$$

where **(0,1]** is an open-closed interval and $\mu_B(a_i)$ is defined by (29). Since α **> 0**, an α-cut subset is formed by the members of **A** that belong to the fuzzy set **B**, such that, $a_i \in \mathbf{B}$ and the **i**-th household poverty index $\mu_B(a_i) \geq \alpha > 0$.

Let **F(α)** stands for the cumulative distribution function by decreasing sizes of the household poverty ratios $\mu_B(a_i)$, $i = 1,...,n$, then $F(\alpha) = P(\mu_B(a_i) \geq \alpha)$. For **F(α)=0.05**, we have

(39) $$\alpha = F^{-1}(0.05) = \max_{\{i\}}\{\mu_B(a_i), \quad s.t., \quad F(\mu_B(a_i)) \geq 0.05\},$$

hence, the fuzzy set $\mathbf{B}_\alpha$ for $\mathbf{F(\alpha)=0.05}$ contains the 5% poorest households, *i.e.*, the 5% greatest values of $\mu_{\mathbf{B}}(\mathbf{a_i})$.

A natural follow-up of the α-cut concept is that of an **r**-nested fuzzy set. It is an **r**-tuple $\mathbf{B} = \{\mathbf{B}_1,...,\mathbf{B}_b,...,\mathbf{B}_r\}$ of ordinary subsets of **A** such that,

$$\mathbf{B_1} \subset ... \subset \mathbf{B_h} \subset ... \subset \mathbf{B_r} \subset \mathbf{B}.$$

Making $\mathbf{B_h} = \mathbf{B_{\alpha h}}$, **h=1,…,r**, it follows that

$$\alpha_1 > ... > \alpha_h > ... > \alpha_r > \mathbf{0}.$$

For **r=2**, $\mathbf{B_1} = \mathbf{B}_\alpha$ may be interpreted as the set of *central* members in **B**, and $\mathbf{B_2} \setminus \mathbf{B}_1 = \mathbf{B} \setminus \mathbf{B}_\alpha$ is the set of *peripheral* ones. Hence, $\mathbf{B}_\alpha$ includes the *core* subset of the poor.

Given a fuzzy set poverty ratio $\mu_\mathbf{B}$**=0.15**, we may choose **r=6**, such that $\alpha_1 = \mathbf{F^{-1}(0.025)}$, *i.e.* $\mathbf{B_1}$ contains the 2.5% poorest households, $\alpha_2 = \mathbf{F^{-1}(0.05)}$, …, and $\alpha_6 = \mathbf{F^{-1}(0.15)}$, *i.e.* the members of $\mathbf{B_6} \subset \mathbf{B}$ are the 15% poorest households.

A policy decision maker should start concentrating his/her attention on the socioeconomic most relevant attributes that the *core* subset $\mathbf{B_1}$ of the poorest 2.5% households are deprived of, purporting the design and activation of an appropriate structural socioeconomic policy. Doing it in a sequential order, *i.e.*, starting with $\mathbf{B_1}$ and following with $\mathbf{B_2}$,…, $\mathbf{B_6}$, the main causes of poverty can be successful abated, if not completely eliminated.

STEP 9. Policy implications. The strong limitation of the univariate analysis and measurement of poverty to inspire sound socioeconomic policies to deal with the causes of poverty were presented in Section 3, Step 8. It cannot go much beyond a policy of transfers according to a principle of eligibility, generally defined as a function of income, as is the case of unemployment insurance, or something more encompassing, as a countercyclical economic policy.

On the other hand, a multidisciplinary analysis of poverty, and its measurement applying fuzzy set theory provides a sound and efficient information set to support the design and activation of structural socioeconomic policies to deal with the main causes of poverty. The relevance of these types of policies depends in an essential way on the relevance of the chosen attributes to represent states of deprivation, social exclusion, and lack of capacity to do or be something.

From the chosen **m** socioeconomic attributes it should be identified those that contribute the most to the household states of poverty with the scope of targeting them in the design of a structural socioeconomic policy. For this, it should be analysed the fuzzy set ratio (37) for each $\mathbf{X_j}$, which offers an estimate of the population deprivation by attribute; furthermore, to the fuzzy set index (29) estimated for each household it should be applied the α-cut and nested fuzzy set definitions.

The concept of structural socioeconomic policy was introduced in Step 6. It necessarily implies the building of a socioeconomic structure capable of generating socioeconomic processes with less unemployment, a higher accumulation of human capital and wealth, a better (less unequal) distribution of both variables, and an increased flow of income with less inequality. The achievement of these objectives requires to deal with issues such as, (*i*) the improvement of the supply of and accessibility to education, which contributes to the human capital formation and its distribution; (*ii*) reorganisation of the financial system; (*iii*) institutional support to small and medium size firms, contributing, jointly with (*ii*), to the accumulation and less unequal distribution of wealth; (*iv*) health services; and (*v*) the building of an advanced socioeconomic infrastructure that should include water supply, sanitary services and electricity. In synthesis, a policy of socioeconomic development having a working and efficient institutional structure which includes the building of an autonomous, independent, efficient, trustworthy and accountable justice system.

The structural policy implications of the multivariate analysis of poverty and the application of the fuzzy set theory to its measurement find support in:

(a) F. D. Roosevelt's historical four freedom statement, *i.e.*, (*i*) freedom of fear, hence, full enhancement of and respect for human rights; (*ii*) freedom of want, *i.e.*, freedom of poverty, deprivation and social exclusion; (*iii*) freedom of speech; and (*iv*) religious freedom.

(b) Francois Perroux's (1969, 1981) definition of economic development as the liberation of all human beings, and of each one of them, the liberation of the whole human being. To accomplish an economic development in the sense of Perroux implies the activation of a dynamic economic process purporting to eradicate the causes of social exclusion and poverty, and to develop the human being capability and space of functioning to reach their full inclusion in the political, economic and social life of a country. Hence, Perroux's definition squarely meets F. D. Roosevelt's four freedom statement.

(c) J. Rawls's Theory of justice (1971), where he advances his maximin concept, *i.e.*, maximising the social welfare of the least advantageous person. α-cut deals with Rawls's maximin criterion in a more comprehensive and social way, offering a highly relevant information set to support the design and implementation of a structural socioeconomic policy. Rawls's maximin criterion has a more limited scope than F. D. Roosevelt's historical statement and F. Perroux's economic development definition.

5. STATISTICAL ANALYSIS OF THE DEGREE OF SIMILARITY BETWEEN THE UA AND MA SETS OF POOR HOUSEHOLDS

This research deals with the univariate and multivariate analyses and measurements of poverty. It presents poverty ratios for both approaches. Therefore, some natural questions arise, such as:

(i) How do they compare in levels? It is not an important issue as far as each approach provides an acceptable measure of poverty. However, in time series, it emerges the important issue of comparability. For this, the time series of poverty ratios should be the outcome of the same method and basic assumptions.

(ii) Do time series of poverty ratios arising from the application of different approaches to the analysis and measurement of poverty follow similar trends and fluctuations? It is a very important issue to be investigated. Should these facts not be observed they would imply that at least one of the methods of measurement and its corresponding basic assumptions should be critically analysed and eventually revised or abandoned.

(iii) Does a given approach, *i.e.*, a given method of analysis and measurement of poverty, provide sound foundations for the design and implementation of socioeconomic policies to abate the structural causes of poverty? For the univariate approach, as discussed in Section 3, Step 8, the answer is **no**; instead, it is **yes** for the multivariate approach that applies the fuzzy set theory, as it was analysed in Section 4, Step 9.

(iv) Do the two methods of analysis and measurement of poverty considered in this study, i.e., UA and MA, define as poor the same subset of households? If not, how much significative is the dissimilarity between the two subsets? The answer to this question will positively contribute to the relative assessment of the two approaches. It can be done with the help of rank correlation analysis. In effect, given a population of **n** households, a poverty ratio **P** as the outcome of either UA or MA, a poverty line $\mathbf{Z(N^*)}$ for households of size $\mathbf{N^*}$, the **i**-th household equivalent income $\mathbf{y_i^e = y_i^e(N^*)}$ introduced in (13), the **i**-th household poverty ratio $\mu_\mathbf{B}(\mathbf{a_i})$ defined in (29), and symbolizing by **[nP]** the greater integer contained in **nP**, we order the **n** households by decreasing values of their fuzzy set poverty ratios $\mu_\mathbf{B}(\mathbf{a_i})$, **i=1,…,n**. Without loss of generality we have,

(40) $$\mu_\mathbf{B}(\mathbf{a_1}) \geq \mu_\mathbf{B}(\mathbf{a_2}) \geq \ldots \geq \mu_\mathbf{B}(\mathbf{a_{[nP]}}) > \mu_\mathbf{B}(\mathbf{a_{[nP]+1}}) \geq \ldots \geq \mu_\mathbf{B}(\mathbf{a_n}),$$

where $\mu_B(a_1)$, being the greatest value of the fuzzy set poverty ratios, corresponds to the poorest household, $\mu_B(a_2)$ to the second poorest, and so on. To the **i**-th household fuzzy set poverty ratio $\mu_B(a_i)$ corresponds the equivalent income $y_i^e = y_i^e(a_i)$. Symbolizing by p_i the rank of $\mu_B(a_i)$ in (40), by π_i the rank of its corresponding equivalent income, and by π_i' the rank of $y^e(a_i)$ in reverse order (countergraduation), we have, for the **n** households:

(41) $$p_i = i, \qquad p_i - \pi_i = i - \pi_i ,$$

$$p_i - \pi_i' = i - \pi_i' = i - n + \pi_i - 1, \qquad i = 1,2,...,n,$$

since, by definition of countergraduation, $\pi_i' = n - \pi_i + 1$.

For any $k \leq n$, *e.g.*, $k = [nP]$, *i.e.*, the subpopulation of poor households determined by a poverty ratio **P**, and applying a rank correlation index, it can be assessed the degree of similarity between the ranks of the **k** poorest households $\mu_B(a_1), \mu_B(a_2), ..., \mu_B(a_k)$ in (40) and the ranks of their corresponding **k** equivalent incomes $y^e(a_1), y^e(a_2), ..., y^e(a_k)$.

Given the pairs of ranks $(p_i, \pi_i) = (i, \pi_i), \quad i = 1,2,...,k,$ the following four rank correlation indexes are considered:

(a) Bravais-Pearson correlation coefficient applied to the ranks (p_i, π_i);

(b) Gini rank correlation (cograduation) index

(42) $$G_c = \left[\sum_i \left| p_i - \pi_i' \right| - \sum_i \left| p_i - \pi_i \right| \right] \Big/ \left[k^2/2 \right]$$

where $\pi_i' = k - \pi_i + 1$ is the reverse rank order of the **i**-th household and $[k^2/2]$ stands for the greatest integer contained in $k^2/2$.

(c) Spearman rank correlation index

(43) $$S = 1 - 6\sum_i (p_i - \pi_i)^2 \Big/ k(k-1)$$

(d) Greiner-Kendall rank correlation index

(44) $$\tau = -1 + 4 \sum_{i<j} d_{ij}^+ \Big/ n(n-1), \qquad d_{ij}^+ = \begin{cases} 1, & \text{if } i < j \text{ and } \pi_i < \pi_j; \\ 0, & \text{otherwise.} \end{cases}$$

for **i,j = 1,2,…,k**.

It follows from (44) that the Greiner-Kendall index is less informative than Bravais-Pearson, Gini and Spearman indexes since the former takes always the value one when $\pi_i < \pi_j$ whatever the size of the difference $\pi_j - \pi_i$, and the value of zero for $\pi_i > \pi_j$, while the latter indexes consider the amount of the differences between the intervening ranks.

(v) Finally, given the **[nP]** poorest households according to the fuzzy set poverty ratios presented in (40) and its corresponding poverty line $\mathbf{Z(N^*)}$, the frequency of equivalent incomes in the **[nP]** poorest fuzzy set poverty ratios greater than or equal to $\mathbf{Z(N^*)}$ divided by **[nP]** offers a measure of the degree of dissimilarity between UA and MA. A value of zero means that UA and MA do not have any poor household in common, *i.e.*, the intersection of the two sets of households is empty. On the other hand, a value of one means that the two sets contain as members the same households. Hence,

(45) $$\mathbf{DIS(MA,UA;P) = (}\text{frequency of } \mathbf{y_i^e \geq Z(N^*)} \text{ in } \mathbf{MA) / [nP]},$$

and for the similarity ratio,

$$\mathbf{SIM(MA,UA;P) = 1 - DIS(MA,UA;P)}.$$

6. A CASE STUDY: ITALY 1993 – 2000

This study deals with the univariate and multivariate analyses and measurements of poverty. Both approaches are applied to Italy, and includes an assessment of their limits, validity and policy implications. The applications cover the years 1993, 1995, 1998 and 2000 and make use of these four years information stored in the Bank of Italy (1995, 1997, 2000, 2002) database. Their corresponding sample sizes are 8089, 8135, 7147 and 8001 households, respectively.

6.1 Univariate approach (UA). The univariate analysis and measurement of poverty follows the methodological research program presented in Section 3.

Step 1. The population object of research is the set **A** of households introduced in (1), corresponding to the Bank of Italy sample surveys. The net total income, *i.e.*, total income minus taxes and social contributions, is the variable considered to determine the state of poverty.

Step 2. The level of net total income is the only indicator considered to determine the households command over resources.

Step 3. The decision rule that partitions the population **A** into poor and non-poor, *i.e.*, having an insufficient or a sufficient command over resources, requires the specification of a poverty line. It is adopted the International Standard of Poverty Line (ISPL), *i.e.*, the poverty line of a two-person household is made equal to the per capita income of the population, hence, $\mathbf{Z(N^*)=Z(2)}$. According to the received terminology, it is a relative poverty line since the income elasticity of ISPL is equal to one. In effect, by definition of ISPL, an **r**% increase of the per capita income will increase ISPL by **r**%.

Step 4. To make the households income comparable among households of different sizes we need to have an equivalence scale. Applying (13), it converts the income of a households of any size into its equivalent income under the assumption that its size is $\mathbf{N^*=2}$. For this, we use the equivalence scale built by Carbonaro (1985, 2002) for the *Commissione di indagine sulla povertà* created by the Presidency of the Counsel of Ministers of the Italian Government. Table 1 presents: (*i*) Carbonaro's equivalence scale **S(N)**, in percentage, for **N=1**, **2**, **3**, **4**, **5**, **6**, **7** and over (**7+**); (*ii*) the estimated values of the ISPL poverty lines in 1000 lire for households of size $\mathbf{N^*=2}$ and for the years 1993, 1995, 1998 and 2000; (*iii*) applying Carbonaro's equivalence scale to ISPL, Table 1 presents the estimated values of the poverty lines for households of sizes $\mathbf{N \neq 2}$.

Step 5. Given the poverty line for a two-person household and for each sample survey, and an equivalence scale we identify the poor from the non-poor households, count them, and register their equivalent disposable incomes.

Steps 6 and 7. Tables 2 and 3 present the estimates of the essential components of a comprehensive poverty ratio discussed in Step 6, Section 3 and the estimates of two comprehensive poverty ratios introduced in Step 7, Section 3. They are: Sen (1976) poverty ratio $\mathbf{P_S}$ specified as a function of **H**, **I** and $\mathbf{G_P}$; and Dagum *et al.* (1992) poverty ratio $\mathbf{P_{DGL}}$ specified as a function of **H**, **I**, $\mathbf{G_p}$, $\mathbf{G_{np}}$ and **D**. Table 2 presents these estimates for Italy 2000 and by size **N**, **N = 1, 2, 3, 4,** and **5+** of the households. Being each subset of households of equal size, the estimates are obtained from the observed disposable incomes. The results will be the same if they were obtained using the equivalent disposable incomes.

Table 3 estimates the essential components and the poverty ratios $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$, for each sample survey. They are obtained after converting each *observed* disposable income into an *equivalent* disposable income for $\mathbf{N^*=2}$.

For each set and subset of households, Tables 2 and 3 include also the estimates of the (observed or equivalent) mean disposable income of the poor ($\mathbf{M_p}$), non-poor ($\mathbf{M_{np}}$) and the total ($\mathbf{M_{tot}}$).

Although $\mathbf{P_S}$ presents significantly low estimates of poverty, Tables 2 and 3 show that $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$ follow a similar pattern by household sizes and for the total population; their time series estimates, although presenting significant differences in levels, show similar trends and fluctuations. The behavioural pattern of these two ratios significantly differ from those of **H**, **I**, **D**, $\mathbf{G_P}$ and $\mathbf{G_{np}}$, asserting their merits and richness among the univariate poverty ratios. This is a natural and expected consequence for $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$ because they incorporate essential and not collinear components such as **H**, **I** and $\mathbf{G_P}$, and **H**, **I, D**, $\mathbf{G_P}$ and $\mathbf{G_{np}}$, respectively. For these reasons the properties advanced for univariate poverty ratios are not discussed because they are relevant only for single argument poverty ratios such as **H** and **I**, or function of **H** *or* **I** as in Kakwani (1980), Thon (1979) and Foster *et al.* (1984), but not for multi-argument poverty ratios such as, $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$, which are functions of several arguments; *e.g.*, the principle of transfer, such that a non-poor household transfers a small amount of income to a poor-household with an income just below the poverty line, such that it becomes non poor and the transferor remains non poor. As a consequence, according to the principle of transfer, **H** and $\mathbf{G_P}$ will decrease, and it is expected that **I**, **D** and $\mathbf{G_{np}}$ may increase. If they counterbalance the decrease of **H** and $\mathbf{G_P}$, the resulting outcome will naturally be an increase of $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$ because of the income level of the poor household receiving the transfer.

The subset of two-person households in Table 2 presents: (*i*) the lowest head-count ratio **H**; and (*ii*) the highest values for the intensity of poverty ratio **I**, the directional income distance ratio between the poor and non-poor disposable income means **D**, and the Gini ratios for the poor ($\mathbf{G_p}$) and the non-poor ($\mathbf{G_{np}}$). These evidences account for the behavioural discrepancy of **H** with respect to both $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$.

For the two-person households, the estimate of **H=7.55**%, besides being the minimum and a very low poverty ratio for the values of **H** in Italy, presents a large contrast with the values of the essential components of the poverty ratios for $\mathbf{N \neq 2}$ because of the extreme poverty of this small frequency of poor two-person households. In effect, this 7.55% poor households have in general very low incomes and present a very high income inequality as substantiated by the values of **I**, **D**, $\mathbf{G_p}$ and $\mathbf{M_p}$ in Table 2. The highest value of **H** corresponds to the **5+** poor-households. It is very high *per-se*, as well as when it is compared with the values of **H** for $\mathbf{N < 5}$. It points out to the high proportion of poverty among households of large sizes.

In Table 3, the time series of **H** is relatively stable while $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$ present wider fluctuations and different trends than **H**. The main reasons for these discrepancies can be explained by the trends and fluctuations of **I**, **D**, $\mathbf{G_p}$ and $\mathbf{M_p}$, where it is evident the large jumps of **I** and $\mathbf{G_p}$ in 1998 and 2000 compared with their values in 1993 and 1995. They offer a sad social evidence of the apparent polarization of income and wealth, and increasing poverty and income inequality generated by the *real globalisation* in action, as opposed to a *social responsible globalisation* of the people, by the people and for the people of the world, regardless of their nationality, residence, religion, and race and ethnic belonging, *i.e.*, without exploitation of the human being by the human being and without exploitation of nations by multinational corporations and other nations, *i.e.*, without imperialism , without corruption, and enhancing in all nations, whatever their relative degrees of development, a justice system that should be independent, autonomous, accountable, professionally competent, and having its members a very high human integrity. Unfortunately, this social responsible globalisation that we ambition is up to now only a *virtual globalisation*.

From the discussion of UA and the results presented in Tables 2 and 3, the univariate analysis and measurement of poverty does not provide any basic information to identify the main causes of poverty for the political decision makers to be able to implement a structural socioeconomic policy to combat deprivation and social exclusion. The most the decision makers can do is to implement an income policy, and in the case of an economic recession, a countercyclical economic policy with the scope of creating employment and reduce the poverty ratios **H**, **I** and **D**, the income inequality of the poor $\mathbf{G_p}$ and to increase the household mean income of the poor, hence, reducing $\mathbf{P_S}$ and $\mathbf{P_{DGL}}$.

A further analysis of UA will be done in the next subsection where we deal with MA.

6.2. Multivariate approach. The multivariate analysis and measurement of poverty follows the methodological research program introduced in Section 4.

Step 1. The set of units object of research is, as in UA, the population of household **A** presented in (1).

Step 2. The main concepts of the multivariate approach to poverty were introduced and analysed in Section 4.

Step 3. The main concepts of poverty together with the content of the Bank of Italy database motivate the choice of a set of socioeconomic attributes or variables that are relevant to assess the state of poverty. The chosen attributes are indicators of each household degree of social exclusion and deprivation, hence, of each household state of poverty. It is represented in (23) by an **m**-order vector.

When the more precise are the **m** chosen attributes to portray the state of poverty, the more accurate is the available statistical information, and the more rigorous and relevant becomes the statistico-mathematical method to translate them into an encompassing poverty ratio. Hence, the more useful this ratio will be to assess the state of poverty, to identify its main causes, and to inspire a sound structural socioeconomic policy to abate the causes of poverty.

Working with the Bank of Italy sample surveys questionnaire, the following 11 simple and composite indicators are selected (several of them consider more than one attribute):

1. Household equivalent disposable income, *i.e.*, total household income minus taxes and social contributions divided by the corresponding value **S(N)** of the equivalence scale;
2. Gender, age and job status of the household head **H**;
3. Educational achievement of the household head **H** and his/her father;
4. Educational achievement of the household spouse **S** and her/his father;
5. Professional occupation of **H**;
6. Household size, number of senior members and job status of **H** and the other household members;
7. Typology and heating services of the household residence;
8. Occupancy title and location of the household residence;
9. Household size and dimension (in square meters) of the household residence;
10. Household size and number of bathrooms in the household residence;
11. Ratio between the number of the household members with income and the household size.

Step 4. The choice of the variable $\mathbf{X_1}$ standing for the household equivalent disposable income requires the application of an equivalence scale. As in UA, Section 6.1, Carbonaro's (1985) equivalence scale is applied to transform each household equivalent disposable income into its equivalent disposable income for a constant household size $\mathbf{N^*=2}$.

For each variable $\mathbf{X_j}$, $\mathbf{j = 1, 2, \ldots, 11}$, *i.e.*, for each simple or composite attribute it is built a table containing the possible simple or composite outcomes of $\mathbf{X_j}$. To each outcome it is assigned a value $\mathbf{x_{ij}}$ defined in (28), where $\mathbf{x_{ij}}$ takes values in the closed unit interval $\mathbf{[0,1]}$. The variable $\mathbf{x_{ij}}$ represents the degree of deprivation, or degree of membership to the fuzzy set $\mathbf{B} \subset \mathbf{A}$, of the **i**-th household, $\mathbf{i = 1, 2, \ldots, n}$, with respect to the **j**-attribute, $\mathbf{j = 1, 2, \ldots, 11}$.

Tables 4 to 14 present the values of $\mathbf{x_{ij}}$ for each possible outcome of $\mathbf{X_j}$, $\mathbf{j = 1, 2, \ldots, 11}$. Table 4 is the only one among these 11 Tables that needs further explanation. For each household $\mathbf{a_i \in A}$, $\mathbf{x_{i1}}$ estimates the degree of deprivation or degree of membership to the fuzzy set $\mathbf{B} \subset \mathbf{A}$ as a function of the **i**-th household equivalent disposable income $\mathbf{y^e_i = y^e(a_i)}$. For this, the following fuzzy set assumptions are introduced:

(i) $\mathbf{x_{i1}=1}$, iff $\mathbf{y^e_i}$ is less than or equal to the $\mathbf{5^{th}}$ percentile, *i.e.*, for all $\mathbf{y^e_i \leq y^e_{0.05} = F^{-1}(0.05)}$;

(ii) $\mathbf{0 < x_{i1} < 1}$, iff $\mathbf{y^e_{0.05} < y^e_i < y^e_{0.25}}$, *i.e.*, $\mathbf{F^{-1}(0.05) < y^e_i < F^{-1}(0.25)}$;

(iii) $\mathbf{x_{i1} = 0}$, iff $\mathbf{y^e_i \geq y^e_{0.25}}$, *i.e.*, for all $\mathbf{y^e_i \geq F^{-1}(0.25)}$.

Assumption (*i*) and (*iii*) are explicit enough and allow $\mathbf{x_{i1}}$ to take its maximum value of **1**, when $\mathbf{y^e_i \leq y^e_{0.05}}$, and its minimum value of **0** when $\mathbf{y^e_i \geq y^e_{0.25}}$. When $\mathbf{y^e_{0.05} < y^e_i < y^e_{0.25}}$, it stands to reason that $\mathbf{x_{i1}}$ will decrease from **1** to **0** as $\mathbf{y^e_i}$ increases from $\mathbf{y^e_{0.05}}$ to $\mathbf{y^e_{0.25}}$. Assuming a linear mathematical path, we have,

(46) $$\mathbf{x_{i1} = a + b\, y^e_i}, \qquad \mathbf{y^e_{0.05} \leq y^e_i \leq y^e_{0.25}},$$

subject to the following two constraints:

(47) $$\mathbf{a + b\, y^e_{0.05} = 1} \qquad \text{and} \qquad \mathbf{a + b\, y^e_{0.25} = 0}.$$

It follows from (46) and (47), that

(48) $$\mathbf{x_{i1} = (y^e_{0.25} - y^e_i) / (y^e_{0.25} - y^e_{0.05})}, \qquad \mathbf{y^e_{0.05} \leq y^e_i \leq y^e_{0.25}}.$$

For the other values of $\mathbf{y^e_i}$, the fuzzy set assumptions (*i*) and (*iii*) apply.

Step 5. The imputed poverty line (26) will be derived for the following fuzzy set poverty ratios μ_B given in (33): $\mu_{B'}$ for 10 out of 11 attributes, where $\mathbf{X_1}$ is excluded; and $\mathbf{\mu_B(X_1)}$ given in (37). These are presented in Table 15 together with the ISPL poverty line and its derived UA poverty ratio **H**, for the four Bank of Italy sample surveys object of inquiry.

Steps 6 and 7. Using the values of $\mathbf{x_{ij} = \mu_B(X_j(a_i))}$ for the 11 chosen attributes given in Tables 4 to 14 and working with the Bank of Italy sample surveys, the Italian fuzzy set poverty ratios for 1993, 1995, 1998 and 2000 are estimated. Hence, the multivariate analysis of poverty, including the social exclusion and Sen's capability and functioning approaches, become operational when the fuzzy set theory is applied as a statistico-mathematical method of poverty measurement.

Table 15 presents the estimated fuzzy set poverty ratios, and when it corresponds, their imputed fuzzy set poverty lines, for the four sample surveys, and Table 16 presents for 2000, the estimates of the fuzzy set poverty ratios and imputed poverty lines for households of size **1, 2, 3, 4** and **5+**. Both Tables present also the corresponding ISPL poverty lines and their derived UA poverty ratios **H**. In synthesis, Tables 15 and 16 present:

(a) Applying (33), the estimates of the fuzzy set poverty ratio μ_B for each population of households for Italy, in 1993, 1995, 1998 and 2000;

(b) Applying (36) and (37), the estimates of the fuzzy set poverty ratio $\mu_B' = \mu_B(\mathbf{j = 2, 3, \ldots, 11})$, and their imputed fuzzy set poverty lines. μ_B' works with **10** indicators (it excludes the equivalent disposable income $\mathbf{X_1}$);

(c) Applying (37), the estimates of the fuzzy set poverty ratio $\mathbf{\mu_B(X_j)}$ for each $\mathbf{X_j}$; $\mathbf{j = 1, 2, \ldots, 11}$. These estimates have very relevant policy implications;

(d) Applying (29), the estimate of the fuzzy set poverty ratio $\mathbf{\mu_B(a_i)}$ for the **i**-th household, $\mathbf{i = 1, 2, \ldots, n}$ is obtained. The weight $\mathbf{w_j}$, $\mathbf{j = 1, 2, \ldots, 11}$, is given by (31) when the statistical information is the outcome of a stratified sample, and by (32) when the sample is purely random, or the information is the outcome of a census. Table 19 presents a selection of **50** estimates of $\mathbf{\mu_B(a_i)}$ and their corresponding equivalent disposable incomes $\mathbf{y^e(a_i)}$ ranked by increasing values of $\mathbf{y^e(a_i)}$; instead, Table 20 presents also a selection of **50** estimates of $\mu_B(a_i)$ and their corresponding values of $\mathbf{y^e(a_i)}$, ranked this time by decreasing values of $\mu_B(a_i)$.

It follows from Table 15 that the time series of the fuzzy set poverty ratio μ_B and μ_B' present similar trend and fluctuations, and for each year, their levels are very close. Their levels are in the neighbourhood of 0.14 and their differences range from **0.0003** to **0.0014**, *i.e.*, the exclusion of $\mathbf{X_1}$ from the fuzzy set poverty ratio does not significantly change either the levels or the direction of μ_B. Besides, the ranks of the time series of μ_B and μ_B' are identical.

The univariate fuzzy set poverty ratio $\mu_B(X_1)$ for the equivalent disposable income $\mathbf{y^e}$ presents an increasing trend, contrasting with the trends of μ_B and μ_B'; besides, the levels of the former are about one percentage point below the latter. When comparing the outcomes of μ_B and μ_B' with the UA poverty ratio, important differences in trends and fluctuations are observed, while $\mu_B(X_1)$ and **H** differ among themselves and with respect to μ_B and μ_B'. It is of interest to observe that there is one year, 1998, in which the ranking of the four ratios coincide; for the other years μ_B and μ_B' also coincide, while $\mu_B(X_1)$ and **H** do not have a single coincident ranking either between themselves or with μ_B and μ_B', which adds further support to the choice of μ_B and μ_B' as poverty ratios.

Table 16 presents the fuzzy set poverty ratios by size of the households in 2000. With some variants, the discrepancies observed for the time series of poverty ratios given in Table 15 are also present in Table 16.

The fuzzy set poverty ratios $\mu_B(X_j)$, $\mathbf{j = 1, 2, \ldots, 11}$, presented in Tables 15 and 16 are highly relevant and informative to identify the main forces that contribute to the state of deprivation of the poor households. Among these 11 attributes, $\mathbf{X_3}$, *i.e.*, the educational achievement of the household head and his/her father, emerges as the most important cause of poverty; the time series values of the poverty ratio $\mu_B(X_3)$ are: **0.52, 0.49, 0.46** and **0.47**. It is followed by $\mathbf{X_9}$, *i.e.*, the household size and dimension (in square meters) of the residence; the time series values of $\mu_B(X_9)$ are: **0.40, 0.39, 0.36** and **0.36**. In the third place comes $\mathbf{X_4}$, *i.e.*, the educational achievement of the household spouse and her/his father; the time series values of $\mu_B(X_4)$ are: **0.38, 0.35, 0.33** and **0.33**. The attribute $\mathbf{X_5}$ is in the fourth place, *i.e.*, the professional occupation of the household head, followed by $\mathbf{X_{10}}$, *i.e.*, the household size and number of bathrooms in the residence.

It is remarkable that, for the four years, the identified five main causes of deprivation and social exclusion that make the highest contribution to the levels of the poverty ratio μ_B, are ranked in the same order. These empirical evidences cogently point out to the priority content of a structural scocioeconomic policy to abate the main causes of poverty, deprivation and social exclusion. For Italy, it should advance a well structured educational and housing policy.

The assessment of the indicators contribution to the state of poverty for the time series (Table 15) is closely confirmed by the estimates reported in Table 16 by household size in 2000. Although the ranks of the causes of poverty by household size are not identical as the ranks of the time series, they have the same main three causes. It is of interest to observe that, for the larger households (size **5+**), the cause that makes the highest contribution to μ_B is the relative dimension of the residence, *i.e.*, $\mathbf{X_9}$.

Step 8. We apply the concepts of α-cuts and nested fuzzy sets to the fuzzy set poverty ratio in Italy 2000, for the total and by household size. Tables 17 and 18 present these and their corresponding weights $\mathbf{w_t}$ for two values of α, *i.e.*, $\mathbf{F(\alpha_1)=0.025}$ and $\mathbf{F(\alpha_2)=0.05}$. Being $\mathbf{n=8001}$, they identify the $\mathbf{[n\ F(\alpha_1)]=200}$ and $\mathbf{[n\ F(\alpha_2)]=400}$ poorest households, respectively. They broadly confirm the poverty ratio ranks by attribute observed in Tables 15 and 16.

As expected, Tables 17 and 18 show a dramatic increase in the fuzzy set poverty ratios. For the poverty ratios by attribute, $\mathbf{\mu_B(X_3)}$ and $\mathbf{\mu_B(X_9)}$ present the maximum values. They correspond to the educational level of the household head and his/her father, and the relative house dimension with respect to the household size. They are over **0.80** for α_1–cut and in the neighbourhood of **0.80** for α_2-cut, adding further support to the identified priority targets of a structural socioeconomic policy.

Step 9. The policy implications of UA and MA were discussed when dealing with the MRP for UA and MA and in their applications.

The univariate analysis and measurement of poverty do not offer any help to the design of a structural economic policy. At most, $\mathbf{P_S}$, $\mathbf{P_{DGL}}$ and their components, can help to implement countercyclical, social assistance and income policies but are unable to identify the main causes of poverty as targets of a structural socioeconomic policy. As discussed before, this scope is accomplished by the multivariate analysis of poverty and the application of the fuzzy set theory to the measurement of poverty, in particular, the fuzzy set poverty ratio by attribute, *i.e.*, $\mathbf{\mu_B(X_j)}$.

Besides the telling arguments offered in support of the multivariate analysis and measurement of poverty, a further one arises from the rank correlation analysis between the fuzzy set poverty ratio $\mathbf{\mu_B(a_i)}$ and its corresponding equivalent disposable income $\mathbf{y^e(a_i)}$, for each $\mathbf{a_i}$, $\mathbf{i = 1, 2, \dots, n}$.

In effect, according to (40) and (41), the **n** households are ordered by decreasing size of $\mathbf{\mu_B(a_i)}$, hence, the rank $\mathbf{p_i}$ of the **i**-th household is $\mathbf{p_i = i}$, $\mathbf{i = 1, 2, \dots, n}$, and the corresponding **i**-th household equivalent disposable income rank is π_i =rank$\mathbf{(y^e(a_i))}$. Therefore, a rank correlation index between $\mathbf{p_i = i}$ and π_i offers an estimate of the degree of similarity or cograduation between the two series of ranks, hence, an estimate of the degree of similarity between UA and MA. The most relevant cases to be estimated are:

(a) The rank correlation of the $[\mathbf{n}\mu_B]$ poorest households, *i.e.*, the correlation between the rank $\mathbf{p_i = i}$ of $\mu_B(\mathbf{a_i})$, and the corresponding rank π_i of $\mathbf{y^e(a_i)}$, where $[\mathbf{n}\mu_B]$ is the greater integer contained in $\mathbf{n}\mu_B$;

(b) The rank correlation as in (a) for the **[nH]** poorest households, where **H** is a UA poverty ratio;

(c) The rank correlation as in (a) for the $[\mathbf{nP_{DGL}}]$ poorest households, where $\mathbf{P_{DGL}}$ is a UA poverty ratio;

(d) The rank correlation as in (a) for the **5%** poorest households;

(e) The rank correlation as in (a) for the **n** households observed in each sample survey.

With the exception of (c) all the other rank correlations were estimated and reported in Table 21. It is not considered the rank correlation for the $[\mathbf{nP_S}]$ poorest households, because the mathematical structure of $\mathbf{P_S}$ gives unacceptable low estimates.

Besides the rank correlation estimates (a)-(e) It can be estimated the measure of dissimilarity **DIS(MA,UA;P)** proposed in (45) and its corresponding measure of similarity **SIM(MA;UA;P) = 1 - DIS(MA,UA;P)**. Hence,

(f) $\mathbf{SIM(MA;UA;P=}\mu_B\mathbf{)}$ is equal to the ratio between the number of equivalent disposable incomes $\mathbf{y^e(a_i) < Z(N^*)}$ and $[\mathbf{n}\mu_B]$, where $\mathbf{Z(N^*) = F^{-1}}(\mu_B)$ is the imputed poverty line derived from μ_B.

For a selection of **50** households, Table 19 presents, for the sample survey in 2000, the ranking of $\mu_B(\mathbf{a_i})$ and $\mathbf{y^e(a_i)}$, *i.e.*, the ranking of the pair $(\mu_B(\mathbf{a_i}), \mathbf{y^e(a_i)})$, by increasing sizes of $\mathbf{y^e(a_i)}$. This Table includes the first 10 households which have the smallest incomes and the last 10 households that have the highest incomes in the observed sample. It is interesting to notice that the household reporting the smallest income has $\mathbf{y^e(a_1) = -121}$ million lire, and the rank of its fuzzy set poverty ratio is rank$(\mu_B(\mathbf{a_1}))$**=2511**. Hence, its poverty ratio $\mu_B(\mathbf{a_1})$ belongs to the **32**nd percentile, which is a non-poor household according to MA because, being μ_B **= 0.1383** the fuzzy set poverty ratio in Italy 2000, the rank of the least poor among the poor households is $[\mathbf{n}\mu_B]$ **= 1106**, which is much less than rank$(\mu_B(\mathbf{a_1}))$**=2511**. Moreover, among the nine households reporting negative incomes, only one belongs to the set of poor households; it is the 8th household ranked by increasing levels of equivalent disposable income, with $\mathbf{y^e(a_8)}$ = **-902300** lire, $\mathbf{\mu_B(a_8)=0.4874}$, and rank$(\mathbf{\mu_B(a_8)})$**=108**. Besides, according to the values of the fuzzy set poverty ratio $\mathbf{\mu_B(a_i)}$, among the 34 households reporting null income, therefore being all of them poor according to any UA poverty ratio stated as a function of income, 8 of them belong to the non-poor households because rank$(\mathbf{\mu_B(a_i)})$ > rank$(\mathbf{\mu_B})$ for the ranks 36 to 43 of the equivalent disposable income $\mathbf{y^e(a_i)}$.

Table 20 completes the information of Table 19. It presents the ranking $\mathbf{p_i = i}$ by decreasing size of the fuzzy set poverty ratio $\mathbf{\mu_B(a_i)}$. The corresponding **i**-th household equivalent disposable income rank is $\pi_i = \pi(\mathbf{y^e(a_i)})$.

It can be verified, from the ranking by decreasing size of $\mathbf{\mu_B(a_i)}$ corresponding to MA, and the ranking by increasing size of $\mathbf{y^e(a_i)}$ corresponding to UA, that among the 50 poorest households according to the $\mathbf{\mu_B(a_i)}$ ranking, only one has an income $\mathbf{y^e(a_i) > Z(N^*)}$, *i.e.*, $\mathbf{y^e(a_{30})=66.6}$ million lire and $\pi(\mathbf{a_{30}})$**=6837**; on the other hand, among the 50 poorest households according to the $\mathbf{y^e(a_i)}$ ranking, 20 of them have a non-poor fuzzy set poverty ratio $\mathbf{\mu_B(a_i)}$, *i.e.*, 20 out of these 50 households have rank$(\mathbf{\mu_B(a_i)})$ > rank$(\mathbf{\mu_B})$. This apparent contrast to identify the poorest 50 households between the MA fuzzy set poverty ratio $\mathbf{\mu_B(a_i)}$ and the UA poverty ratio **H** that considers the income variable only, offers a cogent evidence on the sound bases supporting the fuzzy set poverty ratio specification, and the poor capability of the UA poverty ratios to identify the households suffering deprivation and social exclusion, and *a fortiori*, among the households with null and negative incomes, which are ranked by UA as the poorest households.

Working with the sample survey **n** = 8001, for Italy 2000, Table 21 presents, for selected quantiles of $\mathbf{F(\mu_B(a_i))}$, besides the UA poverty ratio $\mathbf{H = q/n}$ and the MA poverty ratio $\mathbf{\mu_B}$, the following correlation indexes between $\mathbf{p_i}$ and π_i:

(a) Bravais-Pearson correlation coefficient;

(b) Gini rank correlation or cograduation index (42) ;

(c) Spearman rank correlation index (43); and

(d) Greiner-Kendall rank correlation index (44).

Furthermore, Table 21 presents the dissimilarity and similarity ratios introduced in (45).

The four indexes present a similar pattern for a selection of quantiles of $\mathbf{F(\mu_B(a_i))}$, where **F(.)** is the cumulative distribution function with respect to the decreasing values of

$\mu_B(a_i)$. All of them present an almost steady increase with respect to **F**. The rank correlation estimates for the quantile μ_B are:

Bravais-Pearson = 0.44;

Gini = 0.35;

Spearman = 0.47; and

Greiner-Kendall = 0.31.

These rather low estimates of the rank correlation among the **13.83**% poor households identified by the fuzzy set poverty ratio μ_B adds further support to the fuzzy set measurement of poverty. They clearly show that an important subset of households defined as poor for their levels of income are not poor according to the fuzzy set poverty ratios. This conclusion is corroborated by estimates of the similarity and dissimilarity ratios (45) equal to 0.58 and 0.42, respectively, and shown in Table 21.

The estimates of the rank correlation indexes for the whole sample, *i.e.*, **n**=8001, are: 0.67, 0.54, 0.68 and 0.50, respectively.

The levels of the four rank correlation indexes and the similarity ratio between p_i and π_i for Italy in 1993, 1995 and 1998 are very similar to those estimated for Italy in 2000 and presented in Table 21.

Estimating these indexes for the rank of $\mu_B'(a_i)$, *i.e.*, ranking it by decreasing values of the fuzzy set poverty ratio of each household estimated without the attribute X_1, hence, without the equivalent disposable income variable, smaller rank correlation indexes, similar time path and larger dissimilarity ratios for the four sample surveys are obtained, asserting that the MA and UA measurements of poverty are telling different stories.

7. CONCLUSION

The univariate approach (UA) can be traced to the contributions of the late XIX and earlier XX centuries by Booth and Rowntree. It showed strong limitations to represent the states of poverty of those economic units (*e.g.* households) identified as poor. Besides, this univariate approach was unable to consider and incorporate the main causes generating the state of poverty, and even worse, those causes that condition an intergenerational transmission of poverty.

In the 1970s, UA was enriched with Sen poverty ratio, axiomatically derived as a function of the head-count and intensity ratios, and the Gini income inequality ratio of the poors. Sen's ratio was further extended by Dagum, Gambassi and Lemmi with the addition of two new arguments: the directional distance ratio between poor and non-poor subpopulations, and the Gini income inequality of the non-poor. These two arguments took account of the fact that the poor are not a rigid segregated part of a society but they form part of a single society, unequally sharing the same habitat with the non-poor and having an unequal economic relation.

Furthermore, in the 1970s UA was challenged by different multivariate approaches (MA), being the most relevant the French social exclusion and Sen's capability, functioning and entitlement approaches. They provided very sound analytical insights but without proposing a measurement of poverty.

A third MA was done by the UNPD, including two human development indexes for developing and industrialized countries, respectively.

In 1990, Cerioli and Zani brought to the fore a multivariate analysis and measurement of poverty applying the fuzzy set theory. This approach was followed by several authors who enriched MA by offering for the first times a poverty ratio to capture the multidimensional features of poverty.

In this paper, it is further developed the fuzzy set multivariate analysis and measurement of poverty introducing population poverty ratios by attributes and determining their contributions to the total poverty ratio. These ratios provide basic information about the causes of poverty. They are of paramount importance to the design and implementation of a structural socioeconomic policy to abate the causes of poverty. Hence, they purport to break the mechanism responsible for its intergenerational transmission. This policy, being structural, not cyclical, should aim at a process of structural change with the scope of generating stable, efficient and equitable socio-economic processes.

Furthermore, this study applies for the first time the multivariate analysis and measurement of poverty to sample surveys of income and wealth distribution; in this case, those of Italy for 1993, 1995, 1998 and 2000.

Contrary to UA, the MA approach offers fuzzy set poverty ratios for: (*i*) each household; (*ii*) the population of households; and (*iii*) the population of households by attribute. These ratios accurately represent the state of poverty, social exclusion and deprivation of the poor, and clearly identify the causes of poverty by order of importance. The policy implications of these results are straightforward. For the case of Italy, all the sample surveys clearly identified t he following three most important causes of poverty: (*i*) educational level of the household head and his/her father; (*ii*) housing conditions, its dimension and availability of sanitary services relative to the household size; and (*iii*) educational level of the spouse and her/his father. Any structural socio-economic policy to reduce poverty and its intergenerational transmission should definitely start addressing the needs for: (*a*) building efficient and universal equal opportunity access to elementary, technical, professional and scientific educational systems and strongly enhancing the research and development (R&D) capabilities of a nation; and (*b*) developing housing accomodations that warrant healthy living conditions.

The superiority of the fuzzy set poverty measurement with respect to the univariate poverty ratio (based only on household incomes) is supported by the results of a rank correlation analysis. The estimated rank correlation indexes are quite low, indicating that those households identified as poor by MA are significantly different from those given by UA. This result is also confirmed by a similarity index proposed in this study.

The promising results obtained by the application of the fuzzy set theory to the multivariate analysis and measurement of poverty suggest the need of further research on: (i) the specification of alternative weights $\mathbf{w_j}$ for each attribute and the assessment of their relative merits; (*ii*) the modelling of the probability distribution of the fuzzy set poverty ratio by households and comparing it with the distribution of the household equivalent disposable income. Furthermore, an important issue emerging from this study is the need for revisions of sample survey questionnaires such as, income and wealth distributions, household consumption and financial states, labour market, and population censuses, to incorporate new questions purporting to increase the information set for the MA analysis and measurement of poverty.

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